Computer-Aided Control Systems Design Technique with Applications to Aircraft Flying Qualities

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The objective of this paper is to propose a frequency-matching technique for the design of single-input, single-output (SISO) control systems. The parameters of a cascade controller are determined by minimizing a weighted mean square error between the frequency responses of the compensated closed-loop system and a desired closed-loop system. The error is so defined that the controller parameters turn out to be solutions of linear algebraic equations. The technique is applied to the control system design of an aircraft incorporating flying qualities criteria for its longitudinal mode.

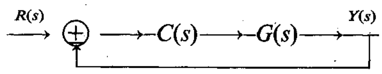
Introduction

IN this paper, a computer-aided control systems design technique is presented that is simple to use and provides a natural tool for designing aircraft control systems that satisfy aircraft flying qualities criteria.[1] Well-known graphical techniques such as root locus, Bode plots, and the Nichol's chart, are quite tedious to work with, particularly, in the case of large order systems. In addition to this, the selection of the controller parameters often depends on the experience and the bias of the designer. The "direct method" of Ragazzini and Franklin[2] is also not very satisfactory. A special adjustment has to be made to make the controller causal. Also, there is no flexibility in picking the order of the controller in the direct method as the controller order is determined by the order of the desired transfer function. In addition to this, the direct method provides no room for introducing the incorporation of any kind of weighting, if needed, whereas in the proposed technique, this can be done easily by multiplying the intergrand in the error by a frequency-dependent weighting function. The idea of using a weighting function to linearize the problem has been in the literature for quite sometime. Originally used by Levy[3] for complex curve fitting, this idea has been used by several investigators[4–6] for model-reduction problems. The technique has also been used for discretization of existing continuous control systems[7,8] where the emphasis was on dealing with hybrid systems. In the present paper, we not only use this powerful idea of linearizing the problem for design purposes in the s–domain but also add several other features in the error criterion to make it a viable and practical design tool. Under the proposed technique, the controller parameters are obtained by minimizing a weighted mean square error between the frequency responses of the compensated closed-loop transfer function and a desired closed-loop transfer function over a certain frequency interval of interest. The desired closed-loop transfer function is synthesized to meet a given set of specifications.

The error is defined in such a way that the controller parameters turn out to be solutions of linear algebraic equations. This is quite significant, as in Refs. 9 and 10, the controller parameters turn out to be solutions of nonlinear algebraic equations that, in general, are harder to solve. As an illustration, we will apply the technique to designing aircraft control systems and show how flying qualities can be directly incorporated into the design process. In this case, the desired transfer function is synthesized so that the aircraft control systems meet the flying quality criteria depending on the aircraft's operating point and mission. Two numerical examples will be presented, one to compare results with Chen and Shieh's technique[10] and the other for the aircraft F-104. Preliminary versions of this paper have appeared in Refs. 11 and 12.

Problem Formulation and the Algorithm

Consider a typical unity feedback control system as represented by its block diagram:



$G(s)$ is the plant, assumed stable, and $C(s)$ is a controller. Suppose

$$F(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} \tag{1}$$

$$G(j\omega) = \frac{L_1(\omega) + jL_2(\omega)}{M_1(\omega) + jM_2(\omega)} \tag{2}$$

$$C(s) = \frac{\sum_{i=0}^{m} a_i s^i}{1 + \sum_{i=1}^{n} b_i s^i} \qquad m \le n \tag{3}$$

where $L_1(\omega)$, $L_2(\omega)$, $M_1(\omega)$, $M_2(\omega)$ are polynomials in ω and a and b are unknown parameters. Suppose $D(s)$ is a desired closed-loop transfer function that has been synthesized to satisfy a given set of specifications. We assume that $D(s)$ has been enhanced, if needed, by the inclusion of nondominant poles, so that $D(s)$ and the compensated closed-loop system $F(s)$ are dimensionally equivalent, i.e., they have the same pole-zero deficiency. The enhancement is desired for a better fit and will be explained later.

Since a stable plant and a stable controller do not necessarily lead to a stable closed-loop system, we impose an additional constraint on the controller parameters to ensure closed-loop stability. Now

$$D_o(s) = \frac{D(s)}{1 - D(s)}$$

is the desired open-loop transfer function corresponding to $D(s)$. Suppose the polar plot of $D_o(s)$ crosses the real axis at $(-r_o, 0)$ corresponding to frequency ω_o. We impose the constraint that the polar plot of $C(s)G(s)$ also crosses the real axis at $(-r_o, 0)$ corresponding to the same frequency ω_o. Nyquist Criterion will then ensure the stability of the compensated closed-loop system.

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Thus, impose the following constraint:

$$(A_o + jB_o) \cdot \frac{\left[\sum_{i=0}^{m} a_i(j\omega_o)^i\right]}{1 + \sum_{i=1}^{n} b_i(j\omega_o)^i} = -r_o + j\,.\,0 \tag{4}$$

where

$$G(j\omega_o) = A_o + jB_o \tag{5}$$

Equating real and imaginary parts in Eq. (4) leads to

$$A_o(a_o - a_2\omega_o^2 + a_4\omega_o^4 - \cdots) - B_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + r_o(1 - b_2\omega_o^2 + \cdots) = 0 \tag{6}$$

and

$$A_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + B_o(a_o - a_2\omega_o^2 + \cdots) + r_o(b_1\omega_o - b_3\omega_o^3 + \cdots) = 0 \tag{7}$$

Note that Eqs. (6) and (7) are linear constraints in controller parameters. Define a mean square error between $F(j\omega)$ and $D(j\omega)$, keeping in view the constraints of Eqs. (6) and (7) as

$$E = \int_{\omega_1}^{\omega_2} \left| \frac{\left[\frac{L_1(\omega) + jL_2(\omega)}{M_1(\omega) + jM_2(\omega)}\right]\left[\frac{\sum_{i=0}^{m} a_i(j\omega)^i}{1 + \sum_{i=1}^{n} b_i(j\omega)^i}\right]}{1 + \left[\frac{L_1(\omega) + jL_2(\omega)}{M_1(\omega) + jM_2(\omega)}\right]\left[\frac{\sum_{i=0}^{m} a_i(j\omega)^i}{1 + \sum_{i=1}^{n} b_i(j\omega)^i}\right]} - \frac{N_1(\omega) + jN_2(\omega)}{P_1(\omega) + jP_2(\omega)} \right|^2 d\omega$$

$$+ \lambda[A_o(a_o - a_2\omega_o^2 + a_4\omega_o^4 - \cdots) - B_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + r_o(1 - b_2\omega_o^2 + \cdots)]$$

$$+ \mu[A_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + B_o(a_o - a_2\omega_o^2 + \cdots) + r_o(b_1\omega_o - b_3\omega_o^3 + \cdots)]$$

$$= \int_{\omega_1}^{\omega_2} \left| \frac{[L_1(\omega) + jL_2(\omega)]\left[\sum_{i=0}^{m} a_i(j\omega)^i\right]}{[M_1(\omega) + jM_2(\omega)]\left[1 + \sum_{i=1}^{n} b_i(j\omega)^i\right] + [L_1(\omega) + jL_2(\omega)]\left[\sum_{i=0}^{m} a_i(j\omega)^i\right]} - \frac{N_1(\omega) + jN_2(\omega)}{P_1(\omega) + jP_2(\omega)} \right|^2 d\omega$$

$$+ \lambda\,[A_o(a_o - a_2\omega_o^2 + a_4\omega_o^4 - \cdots) - B_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + r_o(1 - b_2\omega_o^2 + \cdots)]$$

$$+ \mu[A_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + B_o(a_o - a_2\omega_o^2 + \cdots) + r_o(b_1\omega_o - b_3\omega_o^3 + \cdots)] \tag{8}$$

where $[\omega_1,\omega_2]$ is the frequency interval of interest, λ,μ are Lagrange's undetermined multipliers and

$$A_1 = a_0 - a_2\omega^2 + a_4\omega^4 - \cdots \tag{9a}$$

$$A_2 = a_1\omega - a_3\omega^3 + a_5\omega^5 - \cdots \tag{9b}$$

$$B_1 = 1 - b_2\omega^2 + b_4\omega^4 - \cdots \tag{9c}$$

$$B_2 = b_1\omega - b_3\omega^3 + b_5\omega^5 - \cdots \tag{9d}$$

$$A = L_1(\omega)[P_1(\omega) - N_1(\omega)] - L_2(\omega)[P_2(\omega) - N_2(\omega)] \tag{9e}$$

$$B = L_1(\omega)[P_2(\omega) - N_2(\omega)] + L_2(\omega)[P_1(\omega) - N_1(\omega)] \tag{9f}$$

$$C = N_1(\omega)M_1(\omega) - N_2(\omega)M_2(\omega) \tag{9g}$$

$$D = N_1(\omega)M_2(\omega) + M_1(\omega)N_2(\omega) \tag{9h}$$

$$D(j\omega) = [N_1(\omega) + jN_2(\omega)]/[P_1(\omega) + jP_2(\omega)] \tag{9i}$$

If the controller parameters are obtained by minimizing E, i.e., by equating to zero the partial derivatives of E in Eq. (8) with respect to controller parameters, one gets very complicated nonlinear algebraic equations involving a and b. These equations are hard to solve if a solution exists at all. To get around this difficulty, we modify the error E in Eq. (8) by multiplying the integrand in Eq. (8) by its common denominator. This has, in some sense, a linearizing effect on the minimization of E without losing much accuracy. Intuitively, this is reasonable, as it is based on the fact that if for some rational functions

$$f(x) = \frac{\ell_1(x)}{\ell_2(x)}, \qquad g(x) = \frac{m_1(x)}{m_2(x)}, \qquad f(x) \approx g(x)$$

then $\ell_1(x)m_2(x) \approx \ell_2(x)m_1(x)$ where $\ell_1(x)$, $\ell_2(x)$, $m_1(x)$, $m_2(x)$ are polynomials in some variable x. Thus the modified error is defined as

$$\begin{aligned} E_m = \int_{\omega_1}^{\omega_2} \Bigg| & [L_1(\omega) + jL_2(\omega)]\left[\sum_{i=0}^{m} a_i(j\omega)^i\right][P_1(\omega) + jP_2(\omega)] - [N_1(\omega) + jN_2(\omega)] \\ & \times \left\{[M_1(\omega) + jM_2(\omega)]\left[1 + \sum_{i=1}^{n} b_i(j\omega)^i\right] + [L_1(\omega) + jL_2(\omega)]\left[\sum_{i=0}^{m} a_i(j\omega)^i\right]\right\}\Bigg|^2 \mathrm{d}\omega \\ & + \lambda[A_o(a_o - a_2\omega_o^2 + a_4\omega_o^4 + \cdots) - B_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + r_o(1 - b_2\omega_o^2 + \cdots)] \\ & + \mu[A_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + B_o(a_o - a_2\omega_o^2 + \cdots) + r_o(b_1\omega_o - b_3\omega_o^3 + \cdots)] \end{aligned} \tag{10}$$

$$\begin{aligned} E_m = & \int_{\omega_1}^{\omega_2} \{[AA_1 - BA_2 - CB_1 + DB_2]^2 + [A_1B + AA_2 - DB_1 - CB_2]^2\}\,\mathrm{d}\omega \\ & + \lambda[A_o(a_o - a_2\omega_o^2 + a_4\omega_o^4 - \cdots) - B_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + r_o(1 - b_2\omega_o^2 + \cdots)] \\ & + \mu[A_o(a_1\omega_o - a_3\omega_o^3 + \cdots) + B_o(a_o - a_2\omega_o^2 + \cdots) + r_o(b_1\omega_o - b_3\omega_o^3 + \cdots)] \end{aligned} \tag{11}$$

Equating to zero the partial derivatives of E_m in Eq. (11) with respect to a and b leads to a set of linear algebraic equations. After considerable algebraic manipulation, these equations can be written in the following matrix form:

$$XY = Z \quad \text{or} \quad Y = X^{-1}Z \tag{12}$$

where

$$X = \begin{bmatrix} T_o & 0 & -T_2 & 0 & \cdots & R_1 & S_2 & -R_3 & -S_4 & \cdots & A_0 & B_0 \\ 0 & T_2 & 0 & -T_4 & \cdots & -S_2 & R_3 & S_4 & -R_5 & \cdots & -B_o\omega_o & A_o\omega_o \\ -T_2 & 0 & T_4 & 0 & \cdots & -R_3 & -S_4 & R_5 & S_6 & \cdots & -A_o\omega_o^2 & -B_o\omega_o^2 \\ 0 & -T_4 & 0 & T_6 & \cdots & S_4 & -R_5 & -S_6 & R_7 & \cdots & B_o\omega_o^3 & -A_o\omega_o^3 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ R_1 & -S_2 & -R_3 & S_4 & \cdots & Q_2 & 0 & -Q_4 & 0 & \cdots & 0 & r_o\omega_o \\ S_2 & R_3 & -S_4 & -R_5 & \cdots & 0 & Q_4 & 0 & -Q_6 & \cdots & -r_o\omega_o^2 & 0 \\ -R_3 & S_4 & R_5 & -S_6 & \cdots & -Q_4 & 0 & Q_6 & 0 & \cdots & 0 & -r_o\omega_o^3 \\ -S_4 & -R_5 & S_6 & R_7 & \cdots & 0 & -Q_6 & 0 & Q_8 & \cdots & r_o\omega_o^4 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ A_o & -B_o\omega_o & -A_o\omega_o^2 & B_o\omega_o^3 & \cdots & 0 & -r_o\omega_o^2 & 0 & r_o\omega_o^4 & \cdots & 0 & 0 \\ B_o & A_o\omega_o & -B_o\omega_o^2 & -A_o\omega_o^3 & \cdots & r_o\omega_o & 0 & -r_o\omega_o^3 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{13}$$

$$Y = \begin{bmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ \vdots \\ \lambda \\ \mu \end{bmatrix} \qquad Z = \begin{bmatrix} S_o \\ R_1 \\ -S_2 \\ -R_3 \\ \vdots \\ 0 \\ Q_2 \\ O \\ Q_4 \\ \vdots \\ -r_o \\ 0 \end{bmatrix} \tag{14}$$

and

$$T_m = \int_{\omega_1}^{\omega_2} \omega^m (A^2 + B^2)\, d\omega \qquad (15a)$$

$$R_m = \int_{\omega_1}^{\omega_2} \omega^m (AD - BC)\, d\omega \qquad (15b)$$

$$S_m = \int_{\omega_1}^{\omega_2} \omega^m (AC + BD)\, d\omega \qquad (15c)$$

$$Q_m = \int_{\omega_1}^{\omega_2} \omega^m (C^2 + D^2)\, d\omega \qquad (15d)$$

Thus the matrix Eqs. (12) determine the unknown controller parameters a and b.

Before we give illustrative examples, we wish to make some explanatory comments on the technique.

1) It is assumed that the given plant is stable or, if unstable, it has already been, somehow, stabilized.

2) Observe that the integrand in Eq. (10) is the difference between two polynomials in ω with coefficients involving a and b. Note that when we equate to zero the partial derivatives of E_m *w.r.t.* to a and b, we will get linear algebraic equations in a and b.

3) Since the integrand in E_m is the difference between two polynomials in ω, the best "fit" will be achieved when the polynomials in ω are of the same degree unless the higher degree terms in one of them compared to the other have relatively small coefficients.

4) We can enhance the desired transfer function $D(s)$ so that both polynomials in the integrand of E_m in Eq. (10) are of the same degree. Now

$$D'_o(s) = \frac{D(s)}{1 - D(s)}$$

is the open-loop desired transfer function of a desired unity feedback system. We can add as many relatively nondominant poles to $D'_o(s)$ as needed so that the modified $D'_o(s)$, call it $D''_o(s)$, and $G(s)C(s)$ have the same pole-zero deficiency, i.e.,

(number of poles $G(s)C(s)$ – number of zeros of $G(s)C(s)$)

= (number of poles of $D''_o(s)$ – number of zeros of $D''_o(s)$)

Then multiply $D''_o(s)$ by a suitable gain so that $D_o(s)$ and $D''_o(s)$ have the same steady-rate error constant. We denote this modified $D''_o(s)$ by $D_o(s)$ itself. Then

$$\frac{D_o(s)}{1 + D_o(s)}$$

will be the new modified desired transfer function, which will be used in defining the error E_m. This approach of enhancing the desired transfer function will lead to better minimization of E_m without adding to the order of the controller.

5) For all numerical examples that we have solved using the proposed technique, excepting one, the controller was always stable. Even in the case of this exceptional example (illustrative example number 2 in this paper), when the desired transfer function was enhanced, the controller turned out to be stable. We conjecture that if the desired transfer has reasonable margins of stability and has been enhanced, then the proposed technique will always give a stable controller. This aspect of the technique is under active investigation.

6) The frequency interval of interest $[\omega_1, \omega_2]$ is usually the bandwidth of the desired transfer function and, in general, one gets good results by minimizing E_m over this frequency interval. But, in some numerical examples, if the bandwidth is large, and/or the coefficients in the transfer functions involved are large, then the minimization of E_m over the entire bandwidth leads to large entries in matrices X and Z in Eqs. (13) and (14). This leads to overflow and numerical errors in the matrix inversion, etc. In order to avoid this, it is advisable to minimize E_m over a smaller interval than the bandwidth, usually one-half to one-quarter of the bandwidth. Since the desired transfer function and the compensated system have the same pole-zero deficiency, a "good fit" over a smaller interval usually leads to a good fit over the entire bandwidth. One may have to tinker with the limits of integration for E_m a bit within the above guidelines until satisfactory results are obtained. This should not be very time consuming as the algorithm is computer-aided.

7) Finally, the technique is very easy to use overall. All that the user has to do is to input the order of the controller (one usually starts with the first order), the limits of integration for E_m, and the parameters of the desired transfer function and the plant. As output, the user gets the parameters of the controller and the frequency responses of the desired and the compensated systems. If the fit is "good," one is finished. Otherwise, the user increases the order of the controller and/or alters the limits of integration for E_m as indicated above and continues until satisfactory results are obtained.

Numerical Examples

Two numerical examples are presented. The first one pertains to the design of control systems for the F-104 in its longitudinal directional mode. In the second one, we re-examine the example of Chen and Shieh[10] and compare the controllers designed by the proposed technique and their technique.

Example 1

The pitch rate/(elevator deflection) transfer function of an F-104 at $M = 0.84$ and $h = 30{,}000$ ft is given by

$$\frac{q}{\delta_e} = \frac{-17.8(s + 0.0144)(s + 0.432)s}{(s^2 + 1.12056s + 12.1104)(s^2 + 0.010404s + 0.002601)} \qquad (16)$$

For the short-period mode, we find that the unaugmented system has short-period frequency $\omega_{sp} = 3.48$ rad/sec and short-period damping ratio $\xi_{sp} = 0.161$ that are not satisfactory from the standpoint of flying qualities for this aircraft.[1] Thus, the control system needs augmentation. Using the flying qualities criteria in Ref. 1, we pick

the desired short-period frequency $\omega_{sp} = 4$ rad/s, and

the desired short-period damping ratio $\xi_{sp} = 0.7$ (17)

as the design parameters. From the flying qualities point of view, a desired closed loop transfer function is

$$D(s) = \frac{-4.4(s + 0.432)}{s^2 + 5.6s + 16} \qquad (18)$$

Using Eq. (12) with $\omega_1 = 3.5$ and $\omega_2 = 4.5$, the first-order controller is obtained as

$$C(s) = \frac{0.5456(s + 7.6945)}{s + 16.3645} \qquad (19)$$

Thus, the compensated closed-loop system is

$$\frac{-(9.7125s^4 + 79.0773s^3 + 33.4251s^2 + 0.46495s)}{s^5 + 27.2079s^4 + 109.7097s^3 + 231.9680s^2 + 2.606s + 0.5155} \qquad (20)$$

Using the McFIT program (currently in use by the US military and aerospace industry) to test flying qualities, a second-order equivalent system with 30 equally spaced points over the frequency interval [0.1, 10] was obtained as

$$\frac{-4.659(s + 0.432)}{s^2 + 5.7342s + 13.4834} \text{ with a delay of } -0.0349 \qquad (21)$$

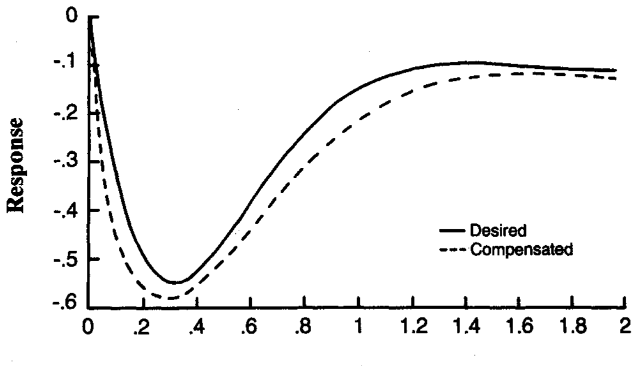

Fig. 1 Time response of the compensated system and the desired system for F-104.

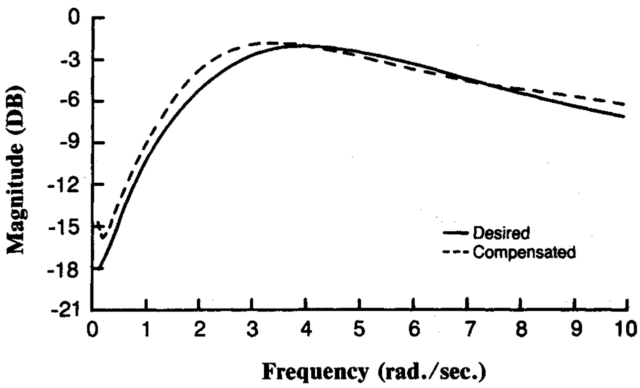

Fig. 2 Magnitude response of the compensated system and the desired system for F-104.

This had a payoff function of 5.057 that is quite satisfactory from the point of view of flying qualities. Thus, the augmented control system designed by the proposed technique satisfies the flying qualities criteria. Figures 1–3 show the comparison of the compensated system and the desired system.

Example 2

The numerical example studied by Chen and Shieh[10] is reexamined for comparison of results. For a unity feedback control system of their example,[10] the open-loop transfer function $G(s)$ is given by

$$G(s) = \frac{6000}{s(s^2 + 40s + 300)} \tag{22}$$

The specifications for the control system design are

$$\begin{aligned} &\text{Velocity error constant} && k_v = 20 \\ &\text{Crossover frequency} && \omega_c = 5 \text{ rad/s} \\ &\text{Damping ratio} && \xi = 0.7 \end{aligned} \tag{23}$$

Chen and Shieh[10] synthesized a second-order desired transfer function satisfying these specifications as

$$D_1(s) = \frac{4.35s + 12.674}{s^2 + 4.984s + 12.674} \tag{24}$$

The second-order controller designed by the proposed technique without enhancing $D_1(s)$ was found to be unsatisfactory, in fact, unstable.[12] Assuming a controller with an equal number of poles and zeros, we note that one polynomial in the integrand of E_m in Eq. (10) for this example will be of two degrees higher than the other polynomial. Also, the coefficients of those higher degree terms are not relatively small. This leads

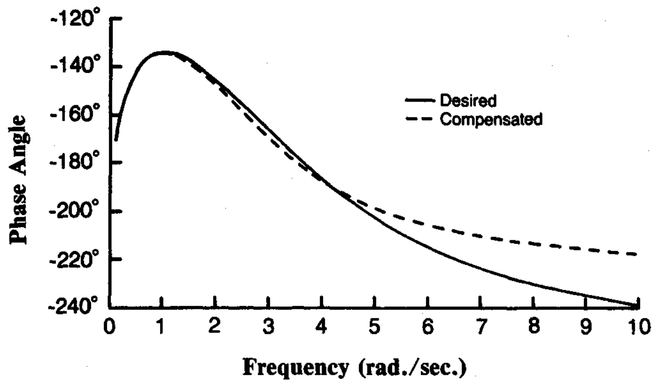

Fig. 3 Phase response of the compensated system and the desired system for F-104.

to a "bad" fit between the two polynomials and hence an unstable controller. The open-loop transfer function corresponding to $D_1(s)$ is

$$D_o(s) = \frac{D_1(s)}{1 - D_1(s)} = \frac{4.35s + 12.674}{s(s + 0.634)} \tag{25}$$

We add two nondominant poles at -40 and -50 to $D_o(s)$ and adjust the gain so that the steady state error constant is unaltered.

Let

$$D_o'(s) = \frac{4.35s + 12.674}{s(s + 0.634)} \times \frac{2000}{(s + 40)(s + 50)} \tag{26}$$

Therefore, the "enhanced" closed-loop desired transfer function is given by

$$D(s) = \frac{D_o'(s)}{1 + D_o'(s)} = \frac{8700s + 25{,}348}{s^4 + 90.634s^3 + 2057.06s^2 + 9968s + 25{,}348} \tag{27}$$

For $D(s)$, we have the following data:

$$\begin{aligned} k_v &= 20 \\ \omega_c &= 4.93 \text{ rad/sec} \\ \xi &= 0.680115 \end{aligned} \tag{28}$$

that are fairly close to the given specifications. Using Eq. (12) with $\omega_1 = 0.0$, $\omega_2 = 1.0$ and $D(s)$ as the desired closed-loop transfer function, a second-order controller is given by

$$C_1(s) = \frac{0.0349s^2 + 0.4446s + 0.9995}{0.0209s^2 + 1.5905s + 1} \tag{29}$$

It is found that the compensated closed-loop system has the following data:

$$k_v = 19.9905 \tag{30a}$$

$$\omega_c = 4.94 \text{ rad/sec} \tag{30b}$$

$$\xi = 0.6807 \tag{30c}$$

Thus, the specifications are fairly closely met.

A second-order controller designed by Chen and Shieh[10] is given by

$$C_2(s) = \frac{s^2 + 10.42s + 20}{s^2 + 32.44s + 20} \tag{31}$$

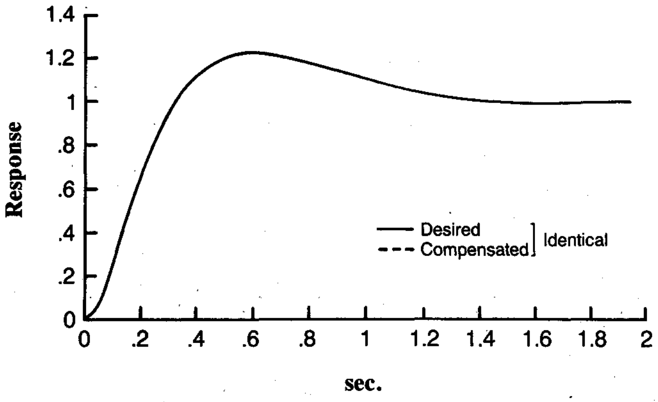

Fig. 4 Time response of the compensated system and the desired system for Example 2.

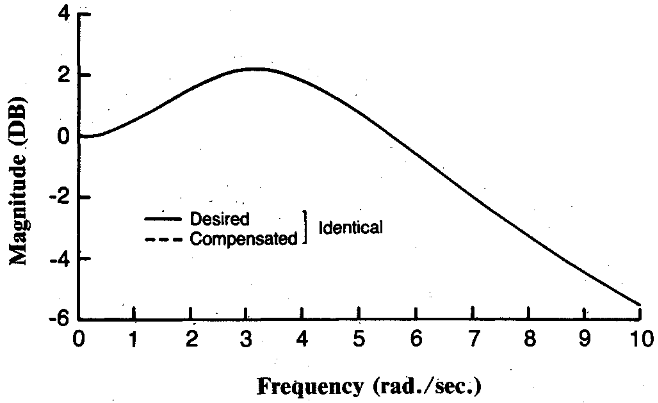

Fig. 5 Magnitude response of the compensated system and the desired system for Example 2.

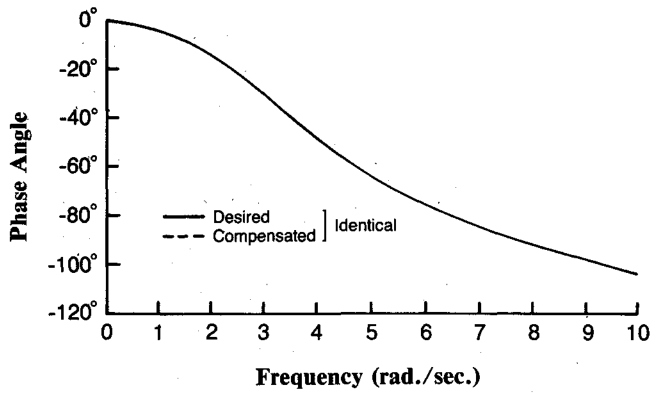

Fig. 6 Phase response of the compensated system and the desired system for Example 2.

The closed-loop system compensated by Chen and Shieh's technique is found to have the following data:

$$k_v = 20 \qquad (32a)$$

$$\omega_c = 5.6 \text{ rad/sec} \qquad (32b)$$

$$\xi = 0.81755115 \qquad (32c)$$

Comparing Eqs. (26) and (28) shows that the results obtained by the proposed technique are slightly better. Figures 4–6 give the comparison of the compensated and the desired systems.

Summary

In this paper, a computer-aided technique for designing a control system is proposed. The controller parameters are obtained by matching the frequency responses of the compensated closed-loop transfer function and a desired transfer function over a certain frequency interval of interest. The technique is easy to simulate as the controller parameters turn out to be the solutions of linear algebraic equations. The effectiveness of the technique is illustrated by its application to the design of aircraft control systems satisfying flying qualities.

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